SET 2013

PAPER – II

MATHEMATICAL SCIENCES

Signature of the Invigilator	Question Booklet No. ....
1.	OMR Sheet No.. ....
Subject Code 04	**ROLL No.**
Time Allowed : 75 Minutes	**Max. Marks : 100**
No. of pages in this Booklet : **11**	No. of Questions : **50**

INSTRUCTIONS FOR CANDIDATES

1. Write your Roll No. and the OMR Sheet No. in the spaces provided on top of this page.
2. Fill in the necessary information in the spaces provided on the OMR response sheet.
3. This booklet consists of fifty (50) compulsory questions each carrying 2 marks.
4. Examine the question booklet carefully and tally the number of pages/questions in the booklet with the information printed above. **Do not accept a damaged or open booklet**. Damaged or faulty booklet may be got replaced within the first **5** minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time given.
5. Each Question has four alternative responses marked (A), (B), (C) and (D) in the OMR sheet. You have to completely darken the circle indicating the most appropriate response against each item as in the illustration.

6. All entries in the common OMR response sheet for Papers I and II are to be recorded in the original copy only.
7. Use only Blue/Black Ball point pen.
8. Rough Work is to be done on the blank pages provided at the end of this booklet.
9. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the spaces allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
10. You have to return the Original OMR Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. **You are, however, allowed to carry the test booklet and the duplicate copy of OMR Sheet** on conclusion of examination.
11. Use of any calculator, mobile phone or log table etc. is strictly prohibited.
12. **There is no negative marking.**

04-13

MATHEMATICAL SCIENCES
PAPER—II

Note :— This paper contains **fifty (50)** objective type questions, each question carrying **two (2)** marks. Attempt **all** the questions.

1. Which one of the following is an algebraically closed field ?
   (A) finite field
   (B) $(\mathbb{Q}, +, \cdot)$
   (C) $(\mathbb{R}, +, \cdot)$
   (D) $(\mathbb{C}, +, \cdot)$

2. Diffusion equation is :
   (A) Hyperbolic
   (B) Elliptic
   (C) Parabolic
   (D) Circular

3. An instructor gives a short quiz involving 10 true-false questions. To test the hypothesis that a student is guessing, the following rules are adopted : (i) if 7 or more answers are correct the student is not guessing; (ii) if less than 7 answers are correct the student is guessing. Then the probability of rejecting the hypothesis when it is correct is :
   (A) $\frac{11}{64}$
   (B) $\frac{13}{64}$
   (C) $\frac{15}{64}$
   (D) $\frac{17}{64}$

4. Which one of the following rings is an integral domain ?
   (A) $\mathbb{Z}_{101}$
   (B) $\mathbb{Z}_{201}$
   (C) $\mathbb{Z}_{2001}$
   (D) $\mathbb{Z}_{91}$

5. If $f(x, y) = \frac{xy}{x+y}$ has continuous first order partial derivatives then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to :
   (A) 1
   (B) f
   (C) 0
   (D) –f

6. The fundamental matrix of the differential equation :
   (A) $\begin{pmatrix} e^{3t} & e^{4t} \\ e^{3t} & 2e^{4t} \end{pmatrix}$
   (B) $\begin{pmatrix} 2e^{3t} & 3e^{4t} \\ e^{3t} & 2e^{4t} \end{pmatrix}$
   (C) $\begin{pmatrix} e^{3t} & 3e^{4t} \\ 2e^{3t} & 2e^{4t} \end{pmatrix}$
   (D) $\begin{pmatrix} e^{3t} & 3e^{4t} \\ e^{3t} & 2e^{4t} \end{pmatrix}$

7. Let the two independent random variables X_1 and X_2 have the same Geometric distribution. Then for any positive integer n, the value of $P(X_1 = r \mid X_1 + X_2 = n)$ is :

(A) $\frac{1}{n+1}$

(B) $\frac{n}{n+1}$

(C) $\frac{r}{n+1}$

(D) $\frac{1}{n+r}$

8. Consider the set $S = \{x \in \mathbb{R} : \sqrt{x}$ is an integer). Then S is :

(A) Finite

(B) Countably infinite

(C) Empty

(D) Uncountable

9. If X be compact and Y be Housdorff topological space then any continuous bijection from X onto Y is a (an) :

(A) Homeomorphism

(B) Homomorphism

(C) Isomorphism

(D) Isometry

10. If L is the Sturm-Liouville operator and $G(x, \xi)$ is the Green's function with $a \le x \le b$, $a \le \xi \le b$, then :

(A) $LG(x, \xi) = \delta(x - \xi)$

(B) $LG(x, \xi) = \delta(x + \xi)$

(C) $LG(x, \xi) = \delta(x)$

(D) $LG(x, \xi) = \delta(\xi)$

11. Let a, b, c be three real numbers. The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{pmatrix}$ is 3, if :

(A) $a \ne b \ne c$ and $a + b + c \ne 0$

(B) $a = b = c$ and $a + b + c \ne 0$

(C) $a \ne b \ne c$ and $a + b + c = 0$

(D) Either $a \ne b \ne c$ or $a + b + c \ne 0$

12. Which property is common to all the following functions ?

(i) $f(z) = \bar{z}$

(ii) $f(z) = \text{Im}(z)$

(iii) $f(z) = \text{Re}(z)$

(iv) $f(z) = \cos(\text{Re}(z)) + i \sin(\text{Re}(z))$

(A) All the four functions are real valued

(B) All the functions are continuous everywhere but differentiable nowhere

(C) All the functions are bounded

(D) All are analytic

13. The necessary and sufficient conditions for two given real numbers a and b to be respectively the mean and variance of some Binomial distribution are :

(A) $a > b > 0$ and $\frac{a^2}{a-b}$ is an integer

(B) $a > b > 0$ and $\frac{b^2}{a-b}$ is an integer

(C) $b > a > 0$ and $\frac{a^2}{a-b}$ is an integer

(D) $b > a > 0$ and $\frac{b^2}{a-b}$ is an integer

14. Let V be a vector space of all real skew-symmetric matrices of order 5. Then dimension of V is :

(A) 5

(B) 10

(C) 20

(D) 25

15. Let X be a Binomial variate with mean 3.75. If X has double mode at X = 3 and X = 4, then its variance is equal to :

(A) 0.3275

(B) 0.8175

(C) 1.8175

(D) 2.8175

16. The order of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 6 & 9 & 7 & 2 & 10 & 1 & 5 & 8 & 4 & 3 \end{pmatrix}$$

is :

(A) 4

(B) 6

(C) 12

(D) 20

17. If $\{a_n\}$ is a monotonically increasing sequence of real numbers, then the sequence $\{a_n\}$:

(A) Always converges

(B) Oscillates infinitely

(C) Always diverges

(D) Converges to its *lim sup*

18. Let A and B be two square matrices of same order. Then which one of the following is not true ?

(A) If A and B are diagonal then A + B is diagonal

(B) If A and B are symmetric then A + B is symmetric

(C) If A and B are skew-symmetric then A + B is skew-symmetric

(D) If A and B are idempotent then A + B is idempotent

19. If q_1, q_2 are generalized coordinates; p_1, p_2 are corresponding generalized momenta and $X = q_1^2 + q_2^2$, $Y = 2p_1 + p_2$, then the Poisson bracket [X, Y] is equal to :

(A) $q_1 + q_2$

(B) $4q_1 + q_2$

(C) $q_1 + 2q_2$

(D) $4q_1 + 2q_2$

20. Let $f, g : [0, 1] \to \mathbb{R}$ be two continuous functions such that $f(0) = g(1) = 0$, $f(1) = g(0) = 1$. Then there exists $x \in (0, 1)$ such that $f(x) = g(x)$. This happens since f, g satisfy :

(A) Heine-Borel property

(B) Neighbourhood property

(C) Intermediate value property

(D) Archemedian property

21. Which of the following is not a convex set ?

(A) $\{(x_1, x_2) : x_1 \geq 0 \text{ and } x_2 \geq 0\}$

(B) $\{(x_1, x_2) : x_1^2 + x_2^2 \leq 3 \text{ and } x_1^2 + x_2^2 \leq 1\}$

(C) $\{(x_1, x_2) : x_1^2 + x_2^2 \leq 3 \text{ and } x_1^2 + x_2^2 \geq 1\}$

(D) $\{(x_1, x_2) : x_1 + 2x_2 \leq 5\}$

22. Let G be a group having elements a and b such that O(a) = 4, O(b) = 2 and $a^3b = ba$. Then O(ab) is :

(A) 2

(B) 5

(C) 6

(D) 4

23. If $(X_1, X_2, ..., X_6)$ is a random sample of size 6 drawn from a normal population with unknown mean μ and standard deviation σ, and

$$t = c\{(X_1 - X_2)^2 + (X_3 - X_4)^2 + (X_5 - X_6)^2\}$$

is an unbiased estimator of σ^2, then the value of c is :

(A) $\frac{1}{6^2}$

(B) $\frac{1}{6^2 - 1}$

(C) $\frac{1}{6}$

(D) None of these

24. The partial differential equation

$x(z - 2y^2)p + y(z - y^2 - 2x^2)q = z(z - y^2 - 2x^2)$,

$\left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right)$ is :

(A) Linear

(B) Semi-linear

(C) Quasi-linear

(D) Nonlinear

25. The function $f(z) = z^n$ $(n \geq 2)$ is conformal :

(A) At no point in $\mathbb{C}$

(B) At every point in $\mathbb{C}$

(C) At every point other than 0 in $\mathbb{C}$

(D) Only at 0

26. The Fredholm integral equation

$$\phi(x) = \lambda \int_a^b K(x, \xi)\, \phi(\xi)\, d\xi$$

has one and only one solution $\phi(x) \equiv 0$ provided that the Fredholm determinant $D(\lambda)$ must be :

(A) Zero

(B) Non-zero

(C) Positive

(D) Negative

27. The quotient field of $\mathbb{Z}[\sqrt{(2)}]$ is :

(A) $\mathbb{R}[\sqrt{(2)}]$

(B) $\mathbb{Q}[\sqrt{(2)}]$

(C) $\mathbb{Z}[\sqrt{(2)}]$

(D) None of these

28. If δ is the central difference operator, then :

(A) $\Delta - \nabla = \delta$

(B) $\Delta + \nabla = \delta$

(C) $\Delta - \nabla = \delta^2$

(D) $\Delta + \nabla = \delta^2$

29. If a continuous random variable X has probability density function $f(x)$ with mean m and standard deviation s = 2, then the random variable Y defined by

$$Y = \int_{-\infty}^{X} f(x)\, dx$$

has a :

(A) Rectangular distribution over [0, 1]

(B) Rectangular distribution over [0, s]

(C) Normal distribution with mean m and standard deviation s

(D) Normal distribution with mean 0 and standard deviation 1

30. In a simple random sampling without replacement, the probability that a specific element is not included in a sample of size n drawn from a population of size N is :

(A) $1 - \frac{n}{N}$

(B) $\frac{n}{N}$

(C) $\frac{N-n}{N-1}$

(D) $\frac{n-1}{N-1}$

31. Which one of the following fields is a splitting field for the polynomial $x^2 + 1$ over $\mathbb{R}$?

(A) $\mathbb{Q}$

(B) $\mathbb{R}$

(C) $\mathbb{C}$

(D) $\mathbb{Z}_2$

32. Let V be the vector space of all real valued functions on $\mathbb{R}$. If $V_1 = \{f \in V : f(-x) = f(x)\}$ and $V_2 = \{f \in V : f(-x) = -f(x)\}$ then :

(A) V_1 is a subspace of V but V_2 is not a subspace of V

(B) V_1 is not a subspace of V but V_2 is a subspace of V

(C) Both V_1 and V_2 are subspaces of V

(D) Neither V_1 nor V_2 is a subspace of V

33. A non constant entire function :

(A) Cannot have an uncountable number of zeros in $\mathbb{C}$

(B) Cannot have a countable number of zeros in a bounded region of $\mathbb{C}$

(C) Cannot have three zeros lying on a straight line

(D) Should have at least one zero in $\mathbb{C}$

34. For a continuous function $\phi(t)$ in $[x_0, x_1]$, if $\int_{x_0}^{x_1} \phi(t)\,\eta(t)\,dt = 0$, for every continuous function $\eta(t)$, then :

(A) $\eta(t) \equiv 0$

(B) $\phi(t) \equiv 0$

(C) $\eta(t) \neq 0$

(D) $\phi(t) \neq 0$

35. Every non-empty subset of $\mathbb{R}$ is :

(A) Order as well as complete

(B) Complete but not necessarily order

(C) Order but not necessarily complete

(D) Neither order nor complete

36. An integer is chosen at random from the set $\{1, 2, 3, \ldots, 200\}$. The probability that the chosen integer is divisible by 3 or 6 or 8 is :

(A) $\frac{81}{200}$

(B) $\frac{41}{100}$

(C) $\frac{83}{200}$

(D) $\frac{21}{50}$

37. The four vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0), (0, 1, a, b) are linearly independent if :

(A) $a \neq 0, b \neq 2$

(B) $a \neq 2, b \neq 0$

(C) $a \neq 0, b \neq -2$

(D) $a \neq -2, b \neq 0$

38. The quadratic form $q(x_1, x_2, x_3) = (x_1 - 2x_2)^2 + x_3^2$ is :

(A) Positive definite

(B) Positive semi-definite

(C) Negative definite

(D) Negative semi-definite

39. Let a population of size N is divided into k strata of sizes $N_1, N_2, \ldots, N_k$. Let in a simple random sampling without replacement, a sample of size n is divided into stratum sample sizes $n_1, n_2, \ldots, n_k$. Then for proportional allocation, the sample size n_i of i^{th} stratum is given by :

(A) $n_i = \frac{n}{N} N_i$

(B) $n_i = \frac{N}{n} N_i$

(C) $n_i = \frac{i}{N} N_i$

(D) $n_i = \frac{i}{n} N_i$

40. Let $f(z) = u + iv$. Then $f(z)$ is analytic everywhere if :

(A) $u = y^2 - x^2, v = 2xy$

(B) $u = 2xy, v = x^2 - y^2$

(C) $u = x^2 - y^2, v = 2xy$

(D) $u = 2xy, v = y^2 + x^2$

41. The condition $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$ for integrability of the exact equation $P(x, y)dx + Q(x, y)dy = 0$ is :

(A) Necessary

(B) Sufficient

(C) Necessary and sufficient

(D) Incorrect condition

42. The order of convergence of iteration in Gauss-Seidal method is :

(A) 4

(B) 3

(C) 2

(D) 1

43. In the complex plane e^z assumes :

(A) Every value excepting zero

(B) Only positive value

(C) Only negative value

(D) Every value

44. A real number X be chosen at random from [1, 49], then the probability of the event

is :

(A) 0.75

(B) 0.5

(C) 0.375

(D) 0.25

45. If g is a continuous function for all $x > 0$ and if $\int_0^x g(t)\,dt = x^2(1+x)$, then the value of g(2) is :

(A) 2

(B) 16

(C) 12

(D) $\frac{1}{2}$

46. By the elimination of arbitrary function *f* from $z = f\left(\frac{xy}{z}\right)$, the partial differential equation obtained is :

(A) $xp(z - yq) = yq(z - px)$

(B) $xq(z - yq) = yp(z - px)$

(C) $yp(z - yq) = xq(z - px)$

(D) $yq(z - yq) = xp(z - px)$

47. For an anharmonic oscillator, the Lagrangian is $L(x, \dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^3 + \beta x \dot{x}^2$, where ω, α, β being constants. If p is the conjugate momentum, then the Hamiltonian H is :

(A) $\frac{2px - p^2}{2(1+2\beta x)} + \frac{1}{2}\omega^2 x^2 + \alpha x^3$

(B) $\frac{2px + p^2}{2(1+2\beta x)} + \frac{1}{2}\omega^2 x^2 + \alpha x^3$

(C) $\frac{2px - p^2}{2(1-2\beta x)} + \frac{1}{2}\omega^2 x^2 + \alpha x^3$

(D) $\frac{2px + p^2}{2(1-2\beta x)} + \frac{1}{2}\omega^2 x^2 + \alpha x^3$

48. Integral curves of Euler's equation are :

(A) Maximals only

(B) Minimals only

(C) Extremals

(D) Nothing else

49. For the equation $x^2 + y^2 + z^2 = a^2$, the number of generalized coordinates is :

(A) 3

(B) 2

(C) 1

(D) 0

50. If the traffic intensity (ρ) is equal to 1, N = 15, and p_n is the probability that there are n customers in M/M/1/N queueing system, then p_n is :

(A) $\frac{1}{2^2}$

(B) $\frac{1}{2^4}$

(C) $\frac{1}{2^6}$

(D) $\frac{1}{2^8}$

ROUGH WORK

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