# SET 2013

## PAPER – III

## MATHEMATICAL SCIENCES

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| Signature of the Invigilator | Question Booklet No. ................................... |
| 1. | OMR Sheet No.. ................................... |
| **Subject Code** 04 | **ROLL No.** |
| **Time Allowed : 150 Minutes** | **Max. Marks : 150** |
| No. of pages in this Booklet : **16** | No. of Questions : **75** |

**INSTRUCTIONS FOR CANDIDATES**

1. Write your Roll No. and the OMR Sheet No. in the spaces provided on top of this page.
2. Fill in the necessary information in the spaces provided on the OMR response sheet.
3. This booklet consists of seventy five (75) compulsory questions each carrying 2 marks.
4. Examine the question booklet carefully and tally the number of pages/questions in the booklet with the information printed above. **Do not accept a damaged or open booklet**. Damaged or faulty booklet may be got replaced within the first **5** minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time given.
5. Each Question has four alternative responses marked (A), (B), (C) and (D) in the OMR sheet. You have to completely darken the circle indicating the most appropriate response against each item as in the illustration.

   (A) (B) ● (D)

6. All entries in the OMR response sheet are to be recorded in the original copy only.
7. Use only Blue/Black Ball point pen.
8. Rough Work is to be done on the blank pages provided at the end of this booklet.
9. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the spaces allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
10. You have to return the Original OMR Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. **You are, however, allowed to carry the test booklet and the duplicate copy of OMR Sheet** on conclusion of examination.
11. Use of any calculator, mobile phone or log table etc. is strictly prohibited.
12. **There is no negative marking.**

**04-13**

# MATHEMATICAL SCIENCES
# PAPER—III

**Note :—** This paper contains **seventy five (75)** objective type questions of **two (2)** marks each. **All** questions are compulsory.

1. Let X be a Poisson variate with parameter m and Y be another discrete random variable whose spectrum is same as X. If the conditional probability of Y for given X is

$$P(Y = r \mid X = x) = \begin{cases} \binom{x}{r} p^r (1-p)^{x-r} & \text{when } r \le x, \\ 0 & \text{when } r > x, \end{cases}$$

where $0 < p < 1$, then :

(A) Y is a Poisson variate with parameter m

(B) Y is a Poisson variate with parameter p

(C) Y is a Poisson variate with parameter mp

(D) Y is a Poisson variate with parameter m + p

2. If $\mathbb{R}_l$ denotes the lower limit topology on $\mathbb{R}$ then which of the following function is continuous from $\mathbb{R}_l$ to $\mathbb{R}_l$ ?

(A) $f(x) = x$

(B) $f(x) = [x]$

(C) $f(x) = \sin x$

(D) None of these

3. The integral equation corresponding to

$$\frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} + e^x y = x; y(0) = 1, y'(0) = -1$$

is :

(A) Fredholm equation of first kind

(B) Volterra equation of first kind

(C) Fredholm equation of Second kind

(D) Volterra equation of Second kind

4. Suppose the spectrum of a random variable X is {0, 1, 2, .....}. If

$$P(X = k + t \mid X \ge k) = P(X = t)$$

for each non-negative integer k and t, then :

(A) X has a Geometric distribution with parameter $p = P(X = 0)$

(B) X has a Poisson distribution with parameter $\lambda = P(X = 0)$

(C) X has a Poisson distribution with parameter $\lambda = P(X \ge 1)$

(D) X has a negative Binomial distribution with parameters $p = P(X = 0)$ and $r = 2$

5. Which one of the following is not a principal ideal domain (PID) ?

(A) $\mathbb{R}[x]$

(B) $\mathbb{Q}[x]$

(C) $\mathbb{Z}[x]$

(D) $\mathbb{Z}[i]$

6. Let $f : [0, 1] \to \mathbb{R}$ be defined as follows :

$$f(x) = \begin{cases} x & \text{when } x \text{ is rational} \\ 1 - x & \text{when } x \text{ is irrational.} \end{cases}$$

Then :

(A) $f$ is continuous everywhere

(B) $f$ is continuous only at $x = \frac{1}{2}$

(C) $f$ is discontinuous at $x = \frac{1}{2}$

(D) $f$ is discontinuous everywhere

7. Number of group homomorphisms from $\mathbb{Z}/4\mathbb{Z}$ to $\mathbb{Z}/6\mathbb{Z}$ is :

(A) 2

(B) 3

(C) 4

(D) 6

8. Using second order Runge-Kutta method with h = 0.1, the value of y(0.2) obtained as a solution of $\frac{dy}{dx} = -xy$, y(0) = 1 is :

(A) 0.9802

(B) 0.9821

(C) 0.9082

(D) 0.9208

9. Let AX = O be a homogenous system of m linear equations in n variables. Then the system of equations has always a non-zero solution if :

(A) m = n

(B) m < n

(C) m > n

(D) none of these

10. Let $X_1, X_2, X_3$, ..... be independent and identically distributed (i.i.d) random variables. Let each $X_i$ assumes only two values $i^k$ and $-i^k$ with equal probability. Then the Weak Law of Large Number (WLLN) can be applied to the sequence of i.i.d. random variables $X_1, X_2, X_3$, ....... if :

(A) $k < \frac{1}{2}$

(B) $k = \frac{1}{2}$

(C) k = 1

(D) $\frac{1}{2} < k < 1$

11. How many times a fair coin must be tossed in order that the probability of the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6 will be at least 0.90 ?

(A) 550

(B) 450

(C) 350

(D) 250

12. The value of the integral $\oint_{|z|=1} \frac{e^{2z}}{\cosh \pi z} dz$ is :

(A) 0

(B) 2i sin 1

(C) 4i sin 1

(D) –4i sin 1

13. The differential equation $k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}, -\infty < x < \infty,$ t > 0 has the solution :

(A) $T(x, t) = \frac{1}{\sqrt{\pi kt}} \exp\left\{-\frac{(x-\xi)^2}{kt}\right\}$

(B) $T(x, t) = \frac{1}{2\sqrt{\pi kt}} \exp\left\{-\frac{(x-\xi)^2}{4kt}\right\}$

(C) $T(x, t) = \frac{1}{\sqrt{\pi kt}} \exp\left\{-\frac{(x+\xi)^2}{kt}\right\}$

(D) $T(x, t) = \frac{1}{2\sqrt{\pi kt}} \exp\left\{-\frac{(x+\xi)^2}{4kt}\right\}$

14. The resolvent kernel of the integral equation with the kernel $K(x, \xi) = 2x$ is :

(A) $xe^{x^2-\xi^2}$

(B) $xe^{\xi^2-x^2}$

(C) $2xe^{x^2-\xi^2}$

(D) $2xe^{\xi^2-x^2}$

15. If $f(z) = \frac{1}{e^z - 1}$, $z \neq 2n\pi i, n = 0, 1, 2, ....,$ then $f$ has :

(A) Removable singularities at the omitted points

(B) Essential singularities at the omitted points

(C) Simple poles at the omitted points

(D) Double poles at the omitted points

16. Let $f(x) = \begin{cases} \sin\frac{1}{x}, & x > 0 \\ x, & x \leq 0 \end{cases}$. Then :

(A) $f$ is continuous everywhere

(B) $f$ has discontinuity of first kind at $x = 0$

(C) $f$ has discontinuity of second kind at $x = 0$

(D) $f$ is monotone

17. Let $|X| = n$ and $V = \{f / f : X \to \mathbb{R}$ is a mapping$\}$. Then dimension of the vector space V over $\mathbb{R}$ is :

(A) n

(B) n !

(C) $n^n$

(D) infinite

18. For the functional $v[y(x)] = \int_0^1 (y^2 + x^2 y')dx$, $y(0) = 0$, $y(1) = a(\neq 1)$, there is :

(A) Maximal

(B) Minimal

(C) Extremal

(D) No extremal

19. The set $S = \{x \in \mathbb{R} : |x - 1| + |x - 2| < 3\}$ is :

(A) Open

(B) Closed

(C) Open and closed

(D) Neither open nor closed

20. For the system $\frac{d^2y}{dx^2} + \lambda y = 0$, $-\pi \leq x \leq \pi$, ($\lambda$ being a parameter), with boundary conditions $y(-\pi) = y(\pi)$ and $y'(-\pi) = y'(\pi)$, when n is a positive integer, the eigen-functions are :

(A) 0, cos nx, sin nx

(B) $e^{nx}$, cos nx, sin nx

(C) $e^{-nx}$, cos nx, sin nx

(D) 1, cos nx, sin nx

21. Let $(X_1, X_2, ....., X_n)$ be a random sample of size $n(>1)$ drawn from a population of a random variable X. Suppose X has a Geometric distribution with parameter p. If

$$t = b\sum_{r=2}^{n} X_{r-1}(aX_r - X_{r-1})$$

is an unbiased estimator of $\frac{1}{p}$, where a and b are two constants independent of p, then :

(A) $a = 1$ and $b = \frac{1}{n-1}$

(B) $a = 2$ and $b = \frac{1}{n-1}$

(C) $a = \frac{1}{n-1}$ and $b = 1$

(D) $a = \frac{1}{n-1}$ and $b = 2$

22. The bilinear transformation $f(z) = \frac{z - i}{z + i}$ maps :

(A) The open upper half plane into the open unit disc

(B) The open unit disc into the open upper half plane

(C) The right half plane into the open unit disc

(D) The left half plane into the open unit disc

23. If 4 dice, each of which has 6 faces marked 1 to 6 are thrown, then the probability of getting a sum 18 on them is :

(A) $\frac{1}{81}$

(B) $\frac{11}{81}$

(C) $\frac{5}{81}$

$\frac{P(z)}{Q(z)}$

(D) $\frac{7}{81}$

24. If the joint density function of the random variables X and Y is

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

then the equation of the least-square regression line of Y on X is :

(A) 2(13y + x) = 15

(B) 2(13y + x) = 17

(C) 2(13y + x) = 19

(D) 2(13y + x) = 21

25. Let T : U $\rightarrow$ V be a surjective linear map and dim U = 6, dim V = 3. Then dim ker T is :

(A) 2

(B) 3

(C) 6

(D) 4

26. The direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$, (a, b, c are constants), are proportional to :

(A) by + cz, cz – ax, ax – by

(B) by – cz, cz + ax, ax – by

(C) by – cz, cz – ax, ax + by

(D) by – cz, cz – ax, ax – by

27. If P(z) and Q(z) are analytic at $\alpha$ with $P(\alpha) \neq 0$ and $\alpha$ is a simple zero of Q(z), then $\alpha$ is a :

(A) Simple pole of $\frac{P(z)}{Q(z)}$ with residue $\frac{P(\alpha)}{Q'(\alpha)}$

(B) Double pole of with residue $\frac{P'(\alpha)}{Q(\alpha)}$

(C) Simple pole of with residue $\frac{P'(\alpha)}{Q(\alpha)}$

(D) Double pole of with residue $\frac{P(\alpha)}{Q'(\alpha)}$

28. Let $A = \{x \in \mathbb{Q} : 2 < x^2 < 3\}$ where $\mathbb{Q}$ denotes the set of rational numbers in $\mathbb{R}$. Then A is :

(A) Compact in $\mathbb{Q}$

(B) Closed and bounded in $\mathbb{R}$

(C) Not compact in $\mathbb{Q}$

(D) Closed and unbounded in $\mathbb{Q}$

29. The dual basis of a basis {(1, 2), (2, 3)} of $\mathbb{R}^2$ is :

(A) {(–3, 2), (2, –1)}

(B) {(1, 3), (2, 2)}

(C) {(3, 2), (2, 1)}

(D) {(2, 1), (3, 2)}

30. If the joint probability mass function of two discrete random variables X and Y is P(X = r, Y = s) = c(2r + s), where r and s can assume all integral values such that $0 \le r \le 2$ and $0 \le s \le 3$, then :

(A)

(B) $c = \frac{1}{21}$ and $P(X = 2, Y = 3) = \frac{1}{6}$

(C) $P(X = 2, Y = 3) = \frac{1}{6}$ and

$P(X \ge 1, Y \le 2) = \frac{3}{7}$

(D) $c = \frac{1}{42}$ and $P(X = 2, Y = 3) = \frac{1}{6}$

31. If a set A has n elements then the number of binary operations defined on A is :

(A) n

(B) $n^2$

(C) n !

(D) $n^{n^2}$

32. For the differential equation $\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0$, where p and q are constants, a non-trivial solution vanishes periodically if :

(A) $p^2 - 4q > 0$

(B) $p^2 - 4q < 0$

(C) $p^2 - 4q \ge 0$

(D) $p^2 - 4q = 0$

33. If u(x) and v(x) are solutions of the self-adjoint equation $\frac{d}{dx}\left\{p(x)\frac{dy}{dx}\right\} + q(x)\,y = 0$, then :

(A) $p\left(u\frac{dv}{dx} - v\frac{du}{dx}\right) = \text{constant}$

(B) $p\left(u\frac{dv}{dx} + v\frac{du}{dx}\right) = \text{constant}$

(C) $q\left(u\frac{dv}{dx} - v\frac{du}{dx}\right) = \text{constant}$

(D) $q\left(u\frac{dv}{dx} + v\frac{du}{dx}\right) = \text{constant}$

34. Let X, Y be independent normal variables with means 6, 7 and variances 9, 16 respectively. If :

$$P(X + Y \le 4\lambda) = P(X - Y \ge 2\lambda)$$

then the value of $\lambda$ is :

(A) 2

(B) 3

(C) 4

(D) 5

35. If $X_1$ and $X_2$ are standard Normal variables with correlation coefficient ρ between them, then :

(A) $V(X_1^2) = V(X_2^2) = 4$

(B) $E(X_1^2) = E(X_2^2) = 4$

(C) the correlation coefficient between $X_1^2$ and $X_2^2$ is $\frac{\rho^2}{2}$

(D) $V(X_1^2) = V(X_2^2) = 2$

36. Upto isomorphism, the number of distinct groups of order 8 is :

(A) 2

(B) 3

(C) 5

(D) 8

37. The particular integral of the equation

$(D^2 - 2aDD' + a^2D'^2)z = e^{ax+y}, \left(D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}\right)$

is :

(A) $-\frac{x}{2}e^{ax+y}$

(B) $\frac{x}{2}e^{ax+y}$

(C) $-\frac{x^2}{2}e^{ax+y}$

(D) $\frac{x^2}{2}e^{ax+y}$

38. Let $A(\subseteq \mathbb{R})$ be a set unbounded above. Then :

(A) For every real number x, there exist $a, b \in \mathbb{R}$ such that $a < x < b$

(B) For every pair of real numbers $x < y$ there is $a \in A$ such that either $a < x$ or $a < y$

(C) For every pair of real numbers $x < y$ there exists $a \in A$ such that $x < a < y$

(D) For every real number x, there exists $a \in A$ such that $x < a$

39. If $f : [a, b] \to \mathbb{R}$ is a monotonic increasing function and if f has a point of discontinuity at a point $x \in [a, b]$, then x is a point of :

(A) Discontinuity of first kind

(B) Discontinuity of second kind

(C) Infinite discontinuity

(D) Removable discontinuity

40. Which one of the following is true ?

(A) UFD ⊂ ED ⊂ PID

(B) PID ⊂ ED ⊂ UFD

(C) ED ⊂ PID ⊂ UFD

(D) PID ⊂ UFD ⊂ ED

41. If $f(x) = k\exp[\{-(9x^2 - 12x + 13)\}], -\infty < x < \infty$, is a probability density function of a random variable X, then :

(A) $k = \frac{3e^9}{\sqrt{\pi}}$ and $V(X) = \frac{1}{3}$

(B) $E(X) = \frac{2}{3}$ and $V(X) = \frac{1}{3}$

(C) $P\left(X > \frac{2}{3}\right) = P\left(X < \frac{2}{3}\right)$

(D) $E(X) = 0$ and $V(X) = \frac{1}{18}$

42. Let $R = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R} \right\}$. Then with usual addition and multiplication of matrices, R is a :

(A) Commutative ring without identity

(B) Non-commutative ring with identity

(C) Division ring but not a field

(D) Field

43. $\int_a^b f(x)\,\delta(x)\,dx = f(0)$ if :

(A) $a \le 0, b > 0$

(B) $a < 0, b > 0$

(C) $a \le 0, b \ge 0$

(D) $a < 0, b \ge 0$

44. Let

$$f(z) = \begin{cases} \dfrac{|z|}{\text{Re}(z)} & \text{when } \text{Re}(z) \neq 0 \\ 0 & \text{when } \text{Re}(z) = 0. \end{cases}$$

Then $f(z)$ :

(A) Has a non-zero limit as $z \to 0$

(B) Is differentiable at $z = 0$

(C) Is continuous but not differentiable at $z = 0$

(D) Is neither continuous nor differentiable at $z = 0$

45. If $X_1$ and $X_2$ be two independent random variables uniformly distributed over the interval [0, 1], then :

(A) $P(X_1 + X_2 < 0.5) = 0.225$

(B) $P(X_1 + X_2 < 0.5) = 0.8755$

(C) $P(X_1^2 + X_2^2 < 0.5) = \dfrac{\pi}{8}$

(D) $P(X_1^2 + X_2^2 < 0.5) = \dfrac{\pi}{4}$

46. Let C[0, 1] be the ring of all real valued continuous functions on [0, 1] and

$$M = \left\{ f \in C[0,1] : f\left(\frac{1}{2}\right) = 0 \right\}.$$ Then M is :

(A) Not a subring of C[0, 1]

(B) A subring but not an ideal of C[0, 1]

(C) An ideal but not a maximal ideal of C[0, 1]

(D) A maximal ideal of C[0, 1]

47. Let $f : \mathbb{C} \to \mathbb{C}$ be analytic such that $f\left(\dfrac{1}{n}\right) = \dfrac{1}{n^2}$, $n = 1, 2, 3, \ldots$, then :

(A) f is a bounded function

(B) there does not exist such a function

(C) $f(z) = z^2, \forall z \in \mathbb{C}$

(D) $f(z) = z, \forall z \in \mathbb{C}$

48. If $T_1$ and $T_2$ are two minimum variance unbiased estimators for a parameter $\theta$ of some given population and $\rho$ is the coefficient of correlation between the random variables $T_1$ and $T_2$, then :

(A) $\rho = 0$

(B) $\rho = 1$

(C) $\rho = -1$

(D) either $\rho = \dfrac{1}{2}$ or $\rho = -\dfrac{1}{2}$

49. Let $A = (a_{ij})_{m \times m}$ be a nilpotent matrix of index n. Then :

(A) $I_m - A$ is invertable

(B) $I_m - A$ is singular

(C) $I_m + A$ is nilpotent

(D) $I_m + A$ is singular

50. Let $f : [a, b] \to \mathbb{R}$ be Riemann integrable over $[a, b]$ and let $F(x) = \begin{cases} \int_a^x f(t)dt, & a < x \le b \\ 0, & x = a. \end{cases}$

Then :

(A) F is continuous on [a, b] and differentiable at the points of continuity of f on [a, b]

(B) F is continuous only at the points of continuity of f on [a, b] and differentiable only at those points where f' exists

(C) F is differentiable everywhere in [a, b]

(D) F is continuous everywhere but differentiable nowhere in [a, b]

51. The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Suppose we want to test whether the average height is greater than 64 inches. If $\mu$ denotes the mean height of the population, then the value of the appropriate statistic under the null hypothesis is $H_0 : \mu = 64$ against the alternative $H_1 : \mu > 64$ is :

$W(\vec{\phi}_1, \vec{\phi}_2, \ldots, \vec{\phi}_n)(t) \neq 0\ \forall\, t \in [a, b]$

$\frac{dx}{dt} = A(t)x,\ x = \begin{pmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{pmatrix}$

(A) 1

(B) 2

(C) 3

(D) 4

52. Let Aut(G) denote the set of all automorphisms of the group G. Then $|\text{Aut}(\mathbb{Z}_p)|$ is :

(A) p

(B) p – 1

(C) 1

(D) $p^2$

53. The series $\sum_{n=1}^{\infty} \tan^{-1} \frac{1}{n^2 + n + 1}$ is :

(A) Convergent

(B) Divergent

(C) Finitely oscillatory

(D) Infinitely oscillatory

54. Let $X_1, X_2, X_3, \ldots$ be i.i.d. random variables with $E(X_i) = m$ and $V(X_i) = \sigma^2$ for all i = 1, 2, 3, ....... If

$$\frac{X_1^2 + X_2^2 + \ldots + X_n^2}{n} \to c \text{ as } n \to \infty$$

in probability, then the value of c is equal to :

(A) $\sigma^2$

(B) $\sigma^2 + \mu^2$

(C) $\frac{\sigma^2}{\sigma^2 + \mu^2}$

(D) $\frac{\mu^2}{\sigma^2 + \mu^2}$

55. If the vector functions $\vec{\phi}_1(t), \vec{\phi}_2(t), \ldots, \vec{\phi}_n(t)$ are the n solutions of the linear homogeneous vector differential equation ,

on the interval $a \le t \le b$, then the solutions are linearly independent iff the Wronskian :

(A) $W(\vec{\phi}_1, \vec{\phi}_2, \ldots, \vec{\phi}_n)(t_0) \neq 0,$ for some $t_0 \in [a, b]$

(B) for some $t_0 \in [a, b]$

(C)

(D)

56. If $[\ ]_{q,\,p}$ denote Poisson bracket, then for three dynamic variables u, v, w, the expression $[uv, w]_{q,\,p}$ is equal to :

(A) $[u, v]_{q,\,p}\, w + w[v, u]_{q,\,p}$

(B) $[u, w]_{q,\,p}\, v + u[w, v]_{q,\,p}$

(C) $[u, w]_{q,\,p}\, v + u[v, w]_{q,\,p}$

(D) $[w, u]_{q,\,p}\, v + u[v, w]_{q,\,p}$

57. For a dynamical system with generalised coordinates q and conjugate momenta p, the Hamiltonian in the case of conservative holonomic system is :

(A) Zero

(B) Constant

(C) Function of q only

(D) Function of p only

58. Let p be the probability of obtaining head in a single toss of a coin. Our aim is to test the hypothesis against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and and $H_0$ is rejected if more than 3 heads are obtained. Then :

(A) Probability of Type I error is $\frac{3}{16}$

(B) Power of the test is $\frac{31}{128}$

(C) Probability of Type I error is $\frac{1}{8}$

(D) Power of the test is $\frac{51}{128}$

59. If the system of equations x = cy + bz, y = az + cx, z = bx + ay has a non-zero solution if :

(A) $a^2 + b^2 + c^2 = 2abc$

(B) $a^2 + b^2 + c^2 + 2abc = 1$

(C) $a^2 + b^2 + c^2 - 2abc = 1$

(D) none of these

60. For $m \neq n$, let $T_1 : \mathbb{R}^n \to \mathbb{R}^m$ and $T_2 : \mathbb{R}^m \to \mathbb{R}^n$ be two linear maps such that $T_2 \circ T_1$ is bijective. Then :

(A) rank $(T_1) = n$; rank $(T_2) = m$

(B) rank $(T_1) = m$; rank $(T_2) = n$

(C) rank $(T_1)$ = rank $(T_2) = m$

(D) rank $(T_1)$ = rank $(T_2) = n$

61. The transversality condition for the functional :

$$v[y(x)] = \int_{x_0}^{x} f(x, y)e^{\tan^{-1} y} \sqrt{1 + y'^2}\, dy;$$

$$f(x, y) \neq 0$$

is :

(A) Hyperbolas

(B) Parabolas

(C) Ellipses

(D) Circles

62. If for a dynamical system with generalised coordinates $q_1, q_2, \ldots, q_n$ and L the Lagrangian, then the energy integral is :

(A)

(B) $\sum_{i=1}^{n} \frac{\partial L}{\partial \dot{q}_i} q_i - L = \text{constant}$

(C) $\sum_{i=1}^{n} \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i - L = \text{constant}$

(D) $\sum_{i=1}^{n} \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i - L = \text{constant}$

63. If X is a continuous random variable with characteristic function exp $\left[-\frac{1}{2}t^2\right]$, then X is a :

(A) Standard Normal variate

(B) Uniform variate over [0, 1]

(C) Gamma variate with parameter $\frac{1}{2}$

(D) Cauchy variate with parameters 0 and 1

64. $\lim_{n\to\infty} \frac{1+\sqrt{2}+\sqrt[3]{3}+.......+\sqrt[n]{n}}{n}$ is :

(A) 1

(B) 0

(C) $\infty$

(D) not determined

65. Let $U=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b=0\right\}$ and $W=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : c+d=0\right\}$ be two subspaces of $M_2(\mathbb{R})$. Then dim(U + W) is :

(A) 2

(B) 3

(C) 4

(D) 6

66. Let P(x, y, z) be a point inside a sphere with centre at the origin. If $r=\sqrt{x^2+y^2+z^2}$, then we have :

(A) $\nabla^2\left(\frac{1}{r}\right)=-2\pi\delta(r)$

(B) $\nabla^2\left(\frac{1}{r}\right)=-4\pi\delta(r)$

(C) $\nabla^2\left(\frac{1}{r}\right)=2\pi\delta(r)$

(D) $\nabla^2\left(\frac{1}{r}\right)=4\pi\delta(r)$

67. If C is the circle | z | = 3 described in the positive sense and if $g(z_0)=\oint_C \frac{2z^2-z-2}{z-z_0}dz$, then the value of g(2) is :

(A) 0

(B) $4\pi i$

(C) $8\pi i$

(D) $16\pi i$

68. The number of optimal solution(s) of the LPP :

Maximize $Z = 6x_1 + 10x_2$

subject to :

$3x_1 + 5x_2 \leq 10$

$5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$

is :

(A) One

(B) Two

(C) Infinite

(D) Finite

69. Let V be a vector space of dimension d and $T : V \to V$ be a linear map with rank r and nullity n. Then :

(A) $rn \le \frac{1}{4}d^2$

(B) $rn > \frac{1}{4}d^2$

(C) d = 4rn

(D) $d^2 < r^2 + n^2$

70. The absolute error in computing $\int_0^1 \frac{dx}{1+x}$ by Simpson's one-third rule with h = 0.25 is :

(A) $1.0672 \times 10^{-1}$

(B) $1.0672 \times 10^{-2}$

(C) $1.0672 \times 10^{-3}$

(D) $1.0672 \times 10^{-4}$

71. The equation for the functional

$$v[z(x, y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial x}\right)^2\right] dxdy$$

is :

(A) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(B) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$

(C) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

(D)

72. If | f(z) | > m on | z | = a, f(z) is regular for | z | ≤ a and | f(0) | < m, then :

(A) f(z) has exactly one zero in | z | < a

(B) f(z) has at least one zero in | z | < a

(C) f(z) has no zero in | z | < a

(D) f(z) is a constant function

73. The canonical transformation whose generating function is

$F_1(q, Q) = q \cos^{-1}(1 - q^2 e^{2Q})^{1/2} + (e^{-2Q} - q^2)$

is :

(A)

(B) $Q = \log\left(\frac{\sin p}{q}\right)$, $P = p \cot q$

(C) $Q = \log\left(\frac{\cos p}{q}\right)$, $P = q \cot p$

(D) $Q = \log\left(\frac{\cos p}{q}\right)$, $P = p \cot q$

74. $\phi(x) = 1 + \lambda x$ is a solution of the integral equation

$$x = \int_0^x e^{x-\xi}\phi(\xi)d\xi, \text{ if :}$$

(A) $\lambda = -1$

(B) $\lambda = 1$

(C) $\lambda \ne \pm 1$

(D) $\lambda = 0$

75. The set S = {a/b : a, b ∈ $\mathbb{Z}$ with b is odd} is a ring under usual addition and multiplication. Then S has :

(A) No maximal ideal

(B) Finitely many maximal ideals

(C) Infinitely many maximal ideals

(D) A unique maximal ideal

## ROUGH WORK

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**ROUGH WORK**

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